Pion production in NEUT

C. Bronner for NEUT authors Kamioka Observatory, ICRR, Tokyo University

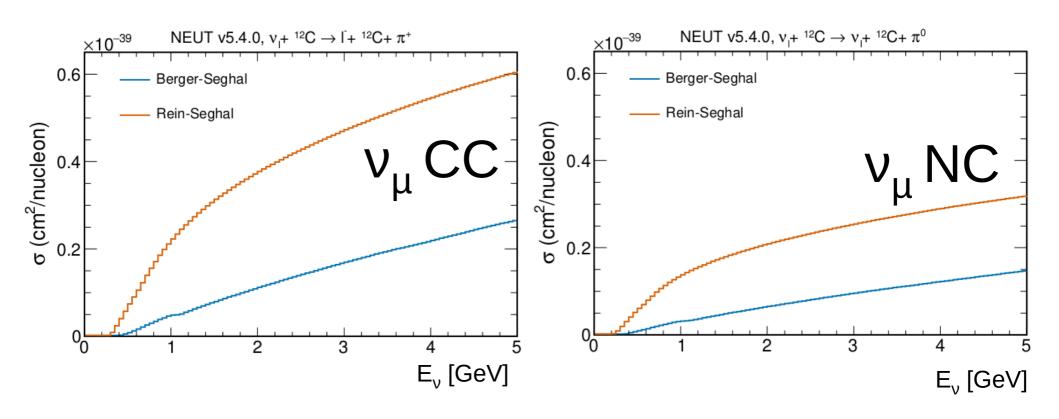


Introduction

- Like most generators, NEUT uses a set of separate models to describe the different interactions modes
- Unless otherwise specified, considering NEUT 5.4.0
- Will describe the implementation for the different modes producing pions:
 - coherent pion production
 - resonant pion production
 - deep-inelastic, including multi-pion background in the resonance region
- Pion Final State Interactions in NEUT
- A few items I think need work:
 - transition from resonant to DIS regions
 - hadronic system in multi-pion mode

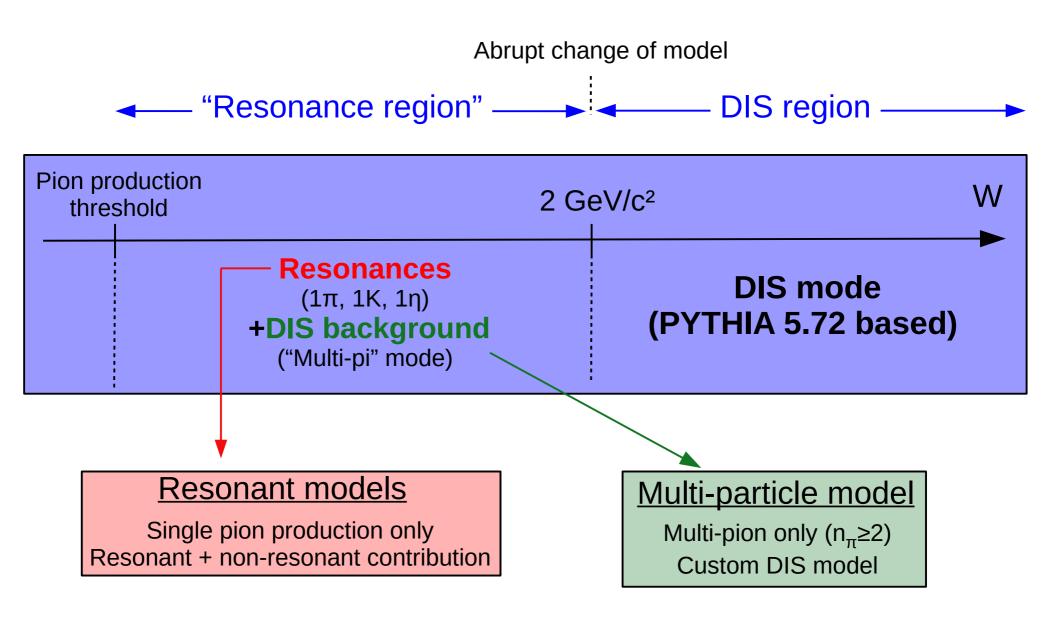
Coherent pion production

- 3 models implemented:
 - Rein-Sehgal (Nucl. Phys. B 223, 29 (1983))
 - Kartavtsev-Paschos (Phys. Rev 74, 054007 (2006))
 - Berger-Sehgal (Phys. Rev. D 79, 053003 (2009))
- Changed default model from RS to BS from NEUT 5.4.0



(Plots by S. Cao, KEK)

Pion production in the resonance region



No $2\pi/3\pi$ resonances No DIS contribution to single pion production below W<2 GeV

Resonant pion production Current implementation

- Based on Rein-Sehgal model (Ann. of Phys. 133(1981),
 Z.Phys.C 35(1987))
- Lepton mass corrections (Phys. Rev. D76, 113004 (2007))
- Includes non-resonant background from RS prescription

As is standard in NEUT, 2 different steps:

- → Pre-calculate cross-section for this mode
- → Actual generation of the events (kinematics)

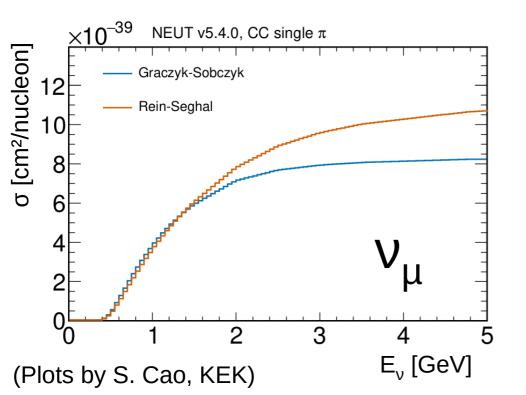
- Cross-section = resonance amplitude x probability to decay into π+baryon
- 18 resonances considered (up to 2 GeV/c²)
 (width, masses, branching ratios, ... not updated recently)
- Integrated over allowed kinematic region in (W,q²)

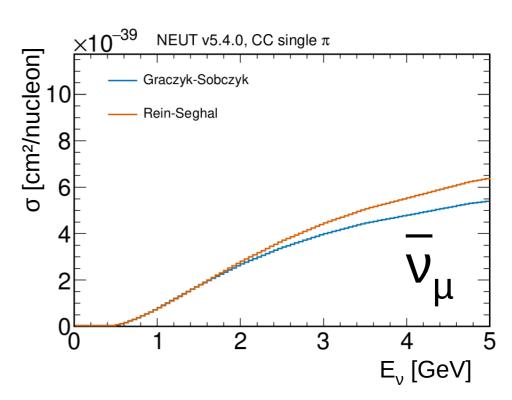
Resonant pion production Current implementation

Can use two different sets of form factors:

- → Standard RS with M_A=1.21 GeV/c² (or 1.11 GeV/c²)
- → Graczyk-Sobczyk

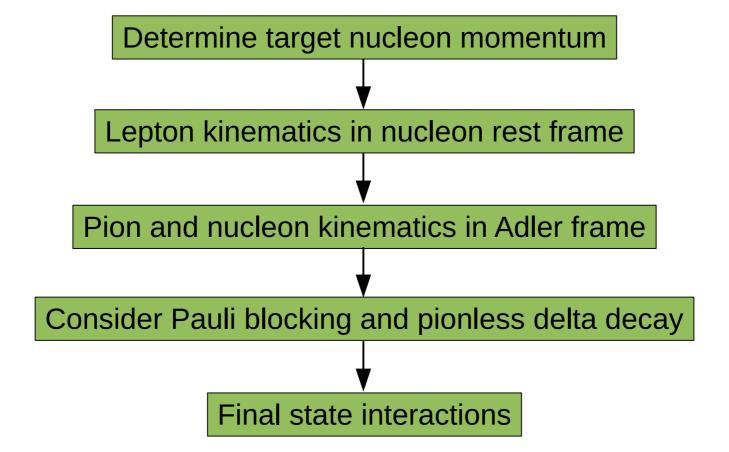
Default is GS with values tuned from a fit of ANL and BNL data (P.Rodrigues et al.): $M_A = 0.95 \text{ GeV/c}^2$, $C_A^{\ 5} = 1.01$, $I_{1/2} = 1.3$





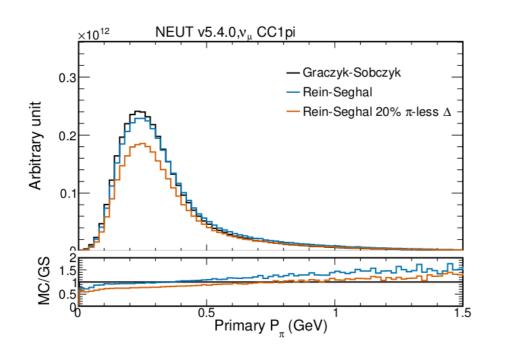
Resonant pion production Event generation

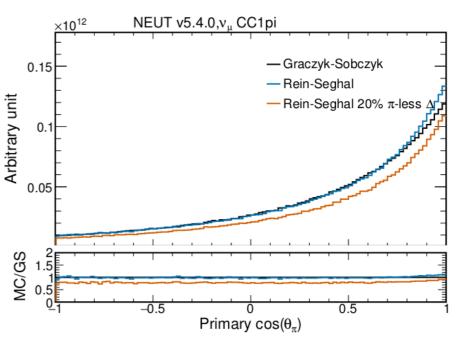
- $^{>}$ Simple nuclear effects: RFG without binding energy and same Fermi momentum as for a CCQE model. No modification of Δ width.
- Pauli blocking implemented for nucleon from delta decay: 2-3% of interactions blocked
- (20% pionless delta decay if 2p2h interactions not enabled)



Resonant pion production Pion direction

- \rightarrow Use RS method for P_{33} (1232)
- Isotropic in Adler's frame for other resonances
- Work by C. Wret to use RS method for the 4 dominant resonances in the future





Pion kinematics for CC single pion modes, ν_{μ} with T2K off-axis flux, by S. Cao

Resonant pion production Plans for the future

Plan to include new model for single pion production in the future:

- → MK model
- → DCC model

- First version of MK model implemented in NEUT (C. Wret)
 - → From NEUT 5.4.1 (not released)
 - → based on Phys. Rev. D 97, 013002 (2018)
- Waiting for additional work by Minoo to be complete to use it as default model
- Implementation of DCC model just started

Multi-pion model Overview

- Multi-pion mode describes the multiparticle ($n_{had} \ge 3$) component in the region 1.3 GeV/c² < W < 2GeV/c²
- Deep-inelastic model with the component n_{had}=2 removed
- In this region PYTHIA cannot be used: custom model
- Assumes that all the events have:
 - one outgoing lepton
 - one outgoing baryon
 - n outgoing pions (n>1)
- No Kaons, η: assume only resonant production for those particles for W<2
 GeV/c²
- Many changes to this mode between NEUT 5.3.4 and 5.3.6: bug fixes + model updates (list in backup)

Multi-pion model Cross-section calculation

- Cross-section for this mode is the standard DIS cross-section, reduced to take into account the fact that we only keep events with n_{had}>3
- \rightarrow Calculated by integrating d² σ /dxdy over possible values of x and y
 - \rightarrow Bjorken x \approx fraction of the nucleon momentum carried by the struck quark
 - → Bjorken y: fraction of the neutrino energy transferred to the hadronic system

 $d^2\sigma/dxdy$ parametrized in terms of structure functions F_1, \dots, F_5

$$\frac{d^2\sigma}{dxdy} \propto \sum_{i=1}^5 \alpha_i \times F_i(x, Q^2)$$

 \rightarrow Use modified Calland-Gross and Albright-Jarlskog relations to relate F_1, F_4, F_5 to F_2 and xF_3

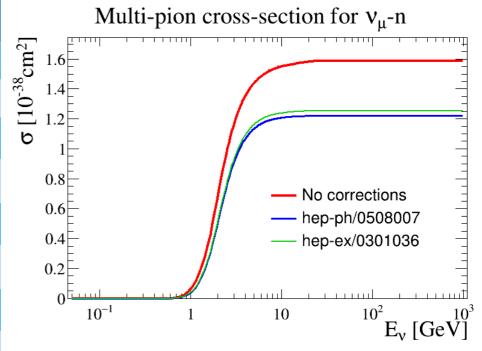
$$F_1(x,Q^2) = \frac{1}{2x} F_2(x,Q^2) \times \left(\frac{1 + 4M^2 x^2 / Q^2}{1 + R(x,Q^2)} \right) \qquad F_4(x,Q^2) = 0 \qquad F_5(x,Q^2) = \frac{F_2(x,Q^2)}{x}$$

 \rightarrow Finally use quark-parton model to compute F_2 and xF_3 from Parton Distribution Functions

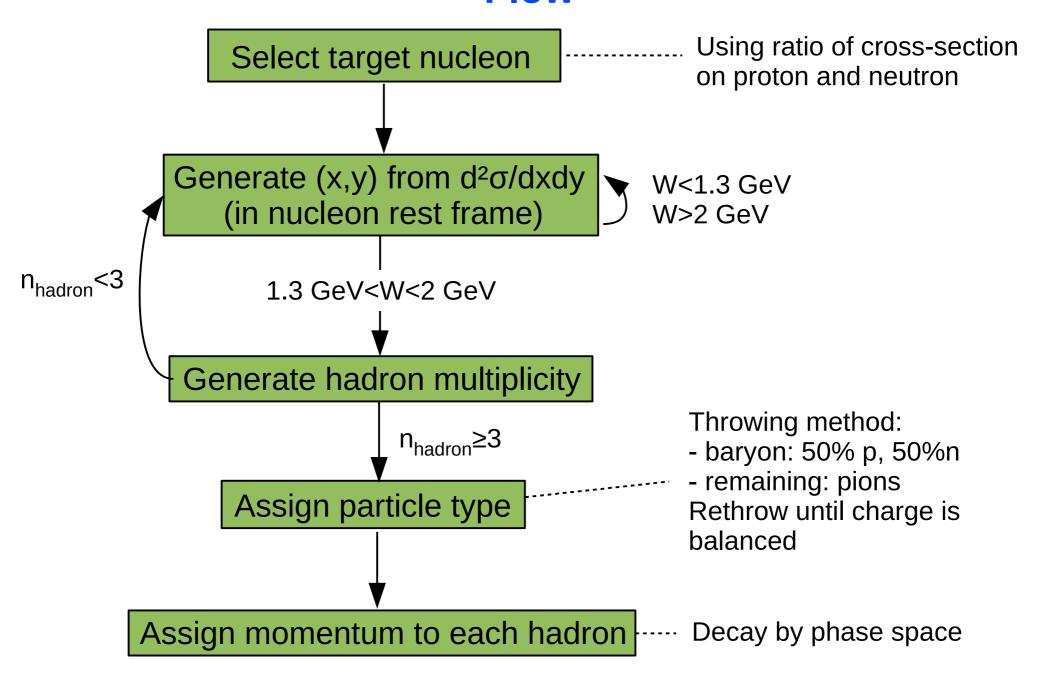
Multi-pion model Bodek-Yang model

- For PDFs, using Bodek-Yang model: GRV98 LO PDFs modified to allow to go to low Q²
- Model with free parameters, determined by a fit of electron scattering and photoproduction data
- Many different version, latest ones not implemented in generators
- Values of the parameters can change significantly between versions, but still give similar predictions

Parameter	hep-ex/0301036 NEUT < 5.3.6	hep-ph/0508007 NEUT ≥ 5.3.6		
Α	0.419	0.538		
В	0.223	0.305		
C _{val1} ^d	0.544	0.202		
C _{val1} u	0.544	0.291		
C _{val2} d	0.431	0.255		
C _{val2} u	0.413	0.189		
C _{sea} d	0.380	0.621		
C _{sea} u	0.380	0.363		

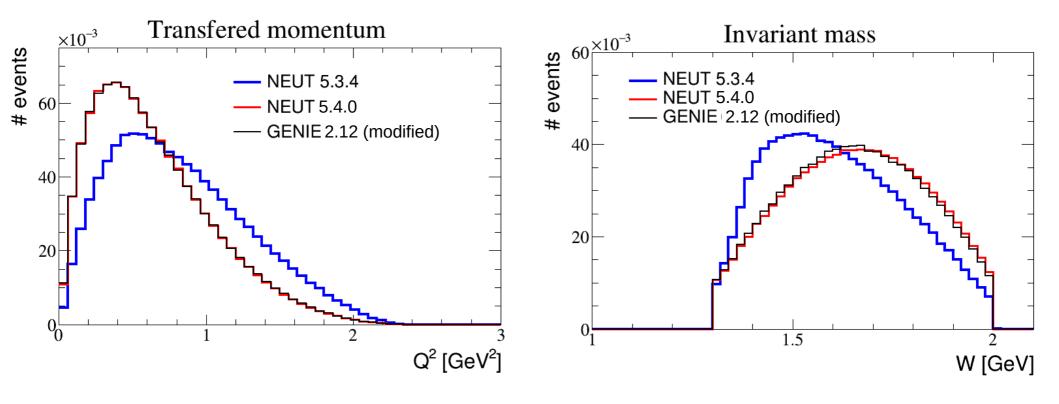


Multi-pion model Flow



Multi-pion model Comparison to GENIE AGKY model

- This mode uses similar method and inputs to generate (x,y)/(W,Q²) to the GENIE low W (AGKY) model
- Prediction of the 2 generators for (W,Q²) agree after the updates from NEUT 5.3.4 to 5.4.0



Note: Need special settings and cuts for generators to match. See talk at Nustec SIS/DIS workshop for details

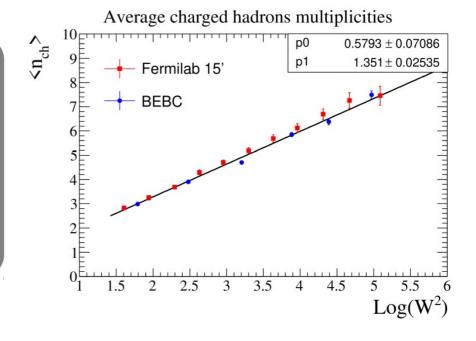
2 GeV neutrinos on free protons Normalized by area

Multi-pion model Multiplicity model

- To determine the number of hadrons produced, use a multiplicity model
- ho Gives the probability to produce a given number of hadrons as a function of W, v/v and target nucleon
- Based on KNO scaling, with free parameters obtained from fits of bubble chamber data

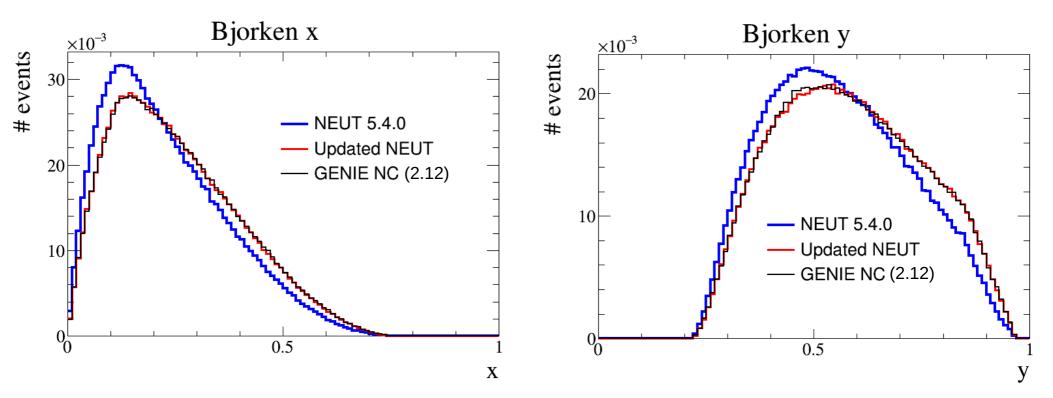
In NEUT 5.4.0, three hadron multiplicity models for the multi-pi mode

- → NE-MULT=0: NEUT default (model used up to now, with a few minor changes)
- → NE-MULT=1: from deuterium bubble chamber fits (CB, hep-ph:1607.06558)
- → NE-MULT=2: AGKY model (GENIE model, hep-ph:0904.4043)



Multi-pion model Neutral current modes

- In 5.4.0, NC multi-pion mode uses CC structure functions (without CKM matrix element), and NC DIS cross-section obtained from CC one
- Initial work on implementing correct NC structure functions.
 With this change, low W NC mode compatible with GENIE
 (2 GeV ν_μ on free neutrons, usual settings to have agreement)
- > Next step will be to compute NC cross-section by integrating $d^2\sigma/dxdy$



Note: here also need special settings and cuts for generators to match.

DIS region

- Pure DIS mode for W>2 GeV/c² based on PYTHIA 5.72
- \triangleright Cross-section calculated by integrating d² σ /dxdy (as for multi-pion mode)
- PYTHIA used for the actual event generation:

CKIN(1) = 0.001

- → input E_v and nucleon four-momentum (from simple RFG model)
- → Loop over PYTHIA event generation until W>2 GeV and right NC/CC type

Modified parameters in PYTHIA 5.72

```
* Lower cut-off on p_t [GeV/c]
   CKIN(5)
             = 0.0001
  Lower CM energy [GeV]
   PARP(2) = 0.001
   Switch to be allowed to decay or not
   MDCY(LUCOMP(111),1) = 0
   MDCY(LUCOMP(221),1) = 0
   MDCY(LUCOMP(311),1) = 0
   MDCY(LUCOMP(223),1) = 0
   MDCY(LUCOMP(130),1) = 0
   MDCY(LUCOMP(310),1) = 0
   MDCY(LUCOMP(331),1) = 1
**** without tau decay(decay at tauola)
   IF(ITAUFLGCORE.eq.1) THEN
    MDCY(LUCOMP(15),1) = 0
   ENDIF
```

Lower edge of allowed sqrt{s} [GeV]

Don't do decays of π^0 , η , K^0 , ω and τ Decay η'

Pion Final State Interactions Overview

- Semi-classical cascade model:
 - pions propagated by steps through the nucleus
 - compute probability to interact at each step from mean free path
- 2 models used depending on pion momentum:
 - " Δ dominated" region p_{π}<500 MeV/c
 - high energy region p_{π} >500 MeV/c
- > Linear transition between the 2 models in the region 400 MeV/c $< p_{\pi} < 500$ MeV/c

Interaction probability at each step depend on nuclear density

$$\frac{\rho(r)}{\rho_0} = \frac{1 + w\frac{r^2}{c^2}}{1 + \exp\left(\frac{r-c}{\alpha}\right)}$$

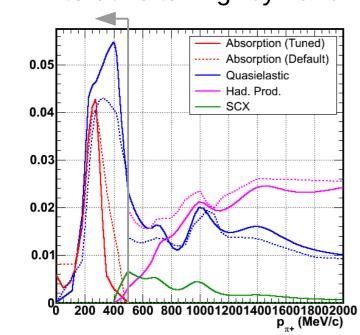
- \sim w=0 for 16 O
- extracted from electron scattering data for others (Jager et al., 1974)
- \sim α ,c also from electron scattering

Pion Final State Interactions Model for p_{π} <500 MeV

- Interaction probability from L.L.Salcedo et al. Nucl. Phys. A484(1998) 79 (valid for 85 MeV<T $_\pi$ <350 MeV extended to 0<p $_\pi$ <500 MeV)
- Absorption probability from E.Oset et al. Nucl. Phys. A468 (1987) 631
- Neglect elastic scattering

Tuning of mean free path

- \checkmark using π^+ -12C data
- Iterative tuning "by hand"

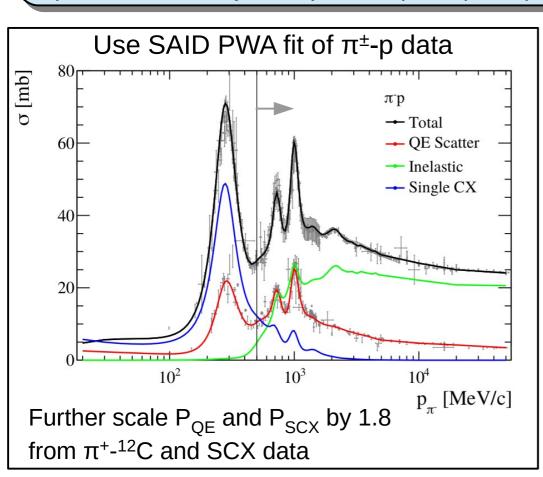


Kinematics (QE):

- > Use the results of phase shift analysis of π -N scattering (Rowe et al., PRC 18(1):584, 1978)
- Medium correction (from R.Seki et al., PRC27 (1983)) is applied to each phase shift
- Single charge exchange included assuming "Δ dominance"

Pion Final State Interactions Model for p_{π} >500 MeV

- Higher energy pions: assume nucleons ~ free nucleons in nucleus
- $^{>}$ Get interaction probablility from π^{\pm} -p data assuming isospin symmetry
 - → π⁺-p ↔ π⁻-n
 - $\rightarrow \pi^--p \leftrightarrow \pi^+-n, \pi^0-n$
- Multiplicity for multi-pion production parameterized from bubble chamber data (J.Whitmore Phys. Rep. 27C (1976) 187)

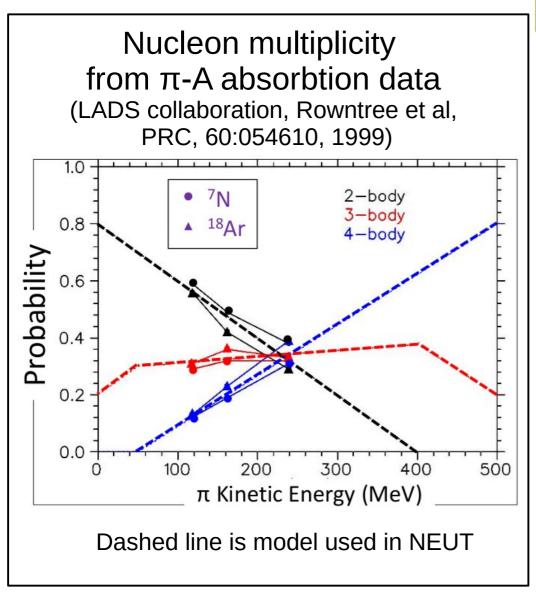


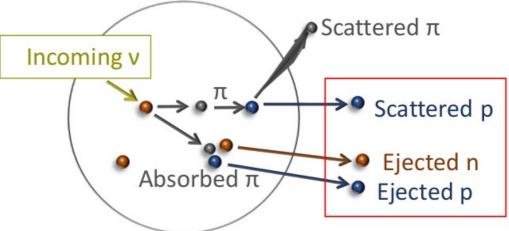
Kinematics

- Multi-pion: by phase space
- Single pion:
 - → Simple elastic-like forward scatter for $T\pi \ge 2$ GeV
 - → Phase shift from SAID PWA for $T\pi$ <400 MeV
 - → Transition between the 2 with quadratic probability between 400 MeV and 2 GeV

Pion Final State Interactions Nucleon emission after pion absorption

By R. Tacik and P. de Perio





Nucleon type:

- From Rowntree et al. for π^+
- Use isospin symmetry for π
- Simple model for π^0

Nucleon kinematics:

- 2-body: follow kinematics of π⁺d → pp, based on B. G. Ritchie PRC 44:533 (1991)
- others: by phase space

Pion Final State Interactions New tuning of interaction strength

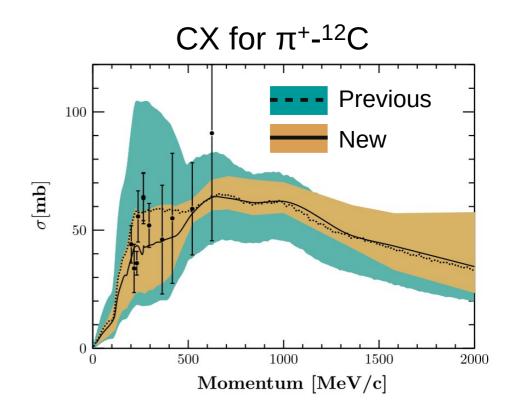
E. S. Pinzon Guerra et al., Phys. Rev. D 99, 052007 (2019) Used in NEUT from 5.4.0

Tune strength of 4 interaction channels:

- → low momentum QE and absorbtion
- → High momentum QE and multi-pion production
- + fraction of charge exchange for low momentum QE

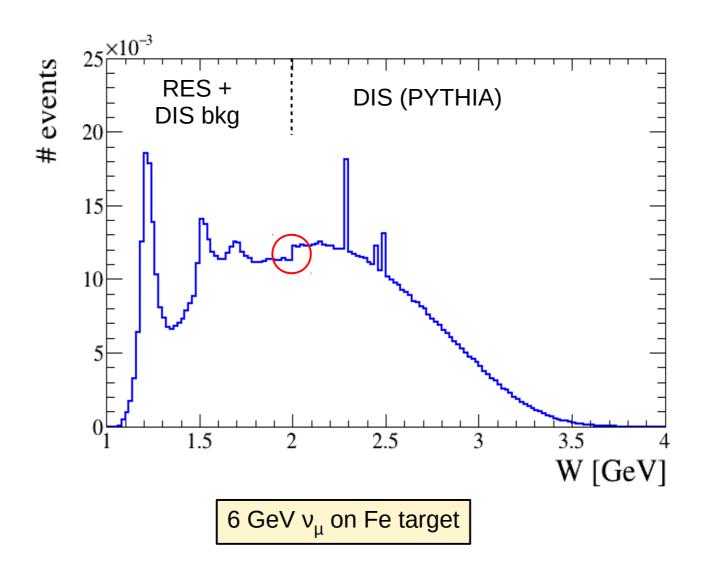
Data on C, O, Al, Fe, Cu, Pb π± beams between 60 MeV and 2 GeV

$\mathbf{Best}\mathbf{fit}\pm1\sigma$					
Carbon-only	Light nuclei	All nuclei			
1.07 ± 0.07	1.08 ± 0.07	1.08 ± 0.07			
1.24 ± 0.05	1.25 ± 0.05	1.26 ± 0.05			
0.79 ± 0.05	0.80 ± 0.04	0.80 ± 0.04			
0.63 ± 0.27	0.71 ± 0.21	0.70 ± 0.20			
2.16 ± 0.34	2.14 ± 0.24	2.13 ± 0.22			
18.36(23)	40.14(40)	53.48(55)			
	Carbon-only 1.07 ± 0.07 1.24 ± 0.05 0.79 ± 0.05 0.63 ± 0.27 2.16 ± 0.34	Carbon-onlyLight nuclei 1.07 ± 0.07 1.08 ± 0.07 1.24 ± 0.05 1.25 ± 0.05 0.79 ± 0.05 0.80 ± 0.04 0.63 ± 0.27 0.71 ± 0.21 2.16 ± 0.34 2.14 ± 0.24			



Transition between RES and DIS regions

See a clear discontinuity when changing models from resonant + bkg to pure DIS at W=2 GeV



Transition between RES and DIS regions

Discontinuity at W=2 GeV/c² hints at a problem, but could be many reasons

- Problem with resonance model?
- (duality not working for ν-A)?
 Problem with multi-pion normalization?
 - Problem with W shape for multi-pion?
 - Problem with W shape from PYTHIA?

- > Pure DIS for W≥2 GeV
- Scheme based on the number of hadrons below:
 - → n_{had}=2: use resonant model
 - → n_{had}≥3: use DIS "multi-pion" model

As a result, multiplicity model matters a lot for multi-pion mode:

- correction factor on the cross-section → normalization of the multi-pion component
- W dependance of $P(n_{had} \ge 3) \rightarrow W$ shape of the multi-pion component

Multi-pion mode Uncertainty on multiplicity model

- Use data from bubble chamber experiments to measure free parameters
- To decorrelate from final state interaction modelisation, use data from hydrogen and deuterium experiments

Author(s), experiment, publ. date	Ref.	Target	W^2 range	Kinematic cuts	Intercept a	Slope b
			$\nu_{\mu} p \rightarrow \mu^{-} X^{+}$	+		
Coffin et al., FNAL E45, 1975	[21]	Н	4-200		1.0 ± 0.3	1.1 ± 0.1
Chapman et al., FNAL E45, 1976	[22]	Н	4-200		1.09 ± 0.38	1.09 ± 0.03
Bell et al., FNAL E45, 1979	[23]	Н	4-100	$Q^2 = 2 - 64 \text{GeV}^2$		1.35 ± 0.15
Kitagaki et al., FNAL E545, 1980	[26]	^{2}H	1-100		0.80 ± 0.10	1.25 ± 0.04
Zieminska et al., FNAL E545, 1983	[27]	^{2}H	4-225		0.50 ± 0.08	1.42 ± 0.03
Saarikko et al., CERN WA21, 1979	[28]	Н	3-200		0.68 ± 0.04	1.29 ± 0.02
Schmitz, CERN WA21, 1979	[29]	Н	4-140		0.38 ± 0.07	1.38 ± 0.03
Allen et al., CERN WA21, 1981	[30]	Н	4-200		0.37 ± 0.02	1.33 ± 0.02
Grässler et al., CERN WA21, 1983	[32]	Н	11-121		-0.05 ± 0.11	1.43 ± 0.04
Jones et al., CERN WA21, 1990	[33]	Н	16-196		0.911 ± 0.224	1.131 ± 0.086
Jones et al., CERN WA21, 1992	[34]	H	9-200		0.40 ± 0.13	1.25 ± 0.04
Allasia et al., CERN WA25, 1980	[35]	^{2}H	2-60		1.07 ± 0.27	1.31 ± 0.11
Allasia et al., CERN WA25, 1984	[38]	^{2}H	8-144	$Q^2 > 1 \mathrm{GeV^2}$	0.13 ± 0.18	1.44 ± 0.06
			$\overline{\nu}_{\mu}p \rightarrow \mu^{+}X$	0		
Derrick et al., FNAL E31, 1976	[14]	Н	4-100	y > 0.1	0.04 ± 0.37	1.27 ± 0.17
Singer, FNAL E31, 1977	[15]	Н	4-100	y > 0.1	0.78 ± 0.15	1.03 ± 0.08
Derrick et al., FNAL E31, 1978	[16]	Н	1-50		0.06 ± 0.06	1.22 ± 0.03
Derrick et al., FNAL E31, 1982	[20]	Н	4-100	0.1 < y < 0.8	-0.44 ± 0.13	1.48 ± 0.06
Grässler et al., CERN WA21, 1983	[32]	Н	11-121		-0.56 ± 0.25	1.42 ± 0.08
Jones et al., CERN WA21, 1990	[33]	Н	16-144		0.222 ± 0.362	1.117 ± 0.141
Jones et al., CERN WA21, 1992	[34]	Н	9-200		-0.44 ± 0.20	1.30 ± 0.06
Allasia et al., CERN WA25, 1980	[35]	^{2}H	7–50		0.55 ± 0.29	1.15 ± 0.10
Barlag et al., CERN WA25, 1981	[36]	^{2}H	6-140		0.18 ± 0.20	1.23 ± 0.07
Barlag et al., CERN WA25, 1982	[37]	^{2}H	6-140		0.02 ± 0.20	1.28 ± 0.08
Allasia et al., CERN WA25, 1984	[38]	^{2}H	8-144	$Q^2 > 1 \mathrm{GeV^2}$	-0.29 ± 0.16	1.37 ± 0.06
			$\nu_{\mu}n ightarrow \mu^{-}X^{-}$	+		
Kitagaki et al., FNAL E545, 1980	[26]	^{2}H	1-100		0.21 ± 0.10	1.21 ± 0.04
Zieminska et al., FNAL E545, 1983	[27]	^{2}H	4-225		-0.20 ± 0.07	1.42 ± 0.03
Allasia et al., CERN WA25, 1980	[35]	^{2}H	2-60		0.28 ± 0.16	1.29 ± 0.07
Allasia et al., CERN WA25, 1984	[38]	² H	8-144	$Q^2 > 1 \mathrm{GeV^2}$	1.75 ± 0.12	1.31 ± 0.04
			$\overline{\nu}_{\mu}n \rightarrow \mu^{+}X$	-		
Allasia et al., CERN WA25, 1980	[35]	^{2}H	7–50		0.10 ± 0.28	1.16 ± 0.10
Barlag et al., CERN WA25, 1981	[36]	^{2}H	4-140		0.79 ± 0.09	0.93 ± 0.04
Barlag et al., CERN WA25, 1982	[37]	^{2}H	2-140		0.80 ± 0.09	0.95 ± 0.04
Allasia et al., CERN WA25, 1984	[38]	^{2}H	8-144	$Q^2 > 1 \mathrm{GeV^2}$	0.22 ± 0.21	1.08 ± 0.06

Many problems:

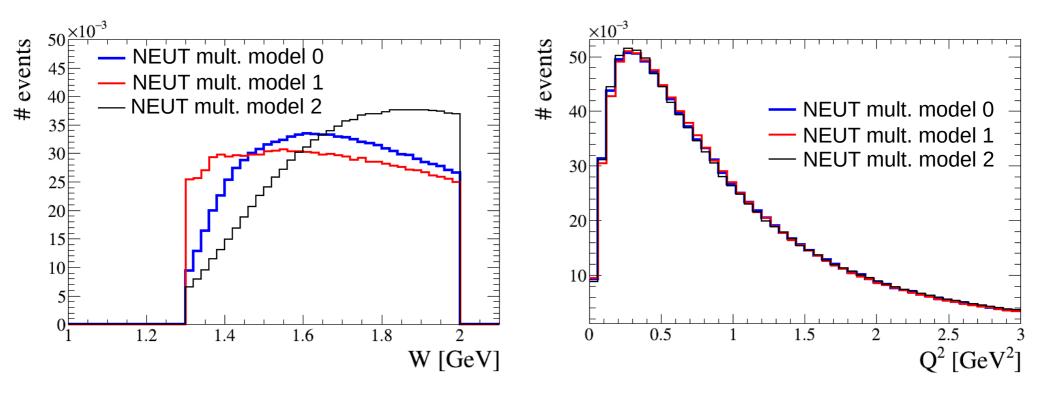
- inconsistent resultsbetween datasets
- actual data hard to find
- no systematic uncertainties most of the time

- NEUT model 0 uses [16] $(\overline{\nu}$ -p) for all types
- GENIE uses [27] for ν and [37] for ν, and symmetry νρ ↔ νη for some parameters

$$< n_{ch}>= a + b \times log(W^2)$$

Multi-pion mode Uncertainty on W distribution

- ' Generate (x,y) based on throwing method, keeping only events with n_{had}≥3
- In multiplicity models, multiplicity probability depends of W $< n_{ch}> = a + b \times log(W^2)$
- Shape of W distribution of the multi-pion component is uncertain as a result of uncertainty on a and b



(T2K near detector flux, area normalized, low W mode W<2 GeV, $n_{\pi} \ge 2$)

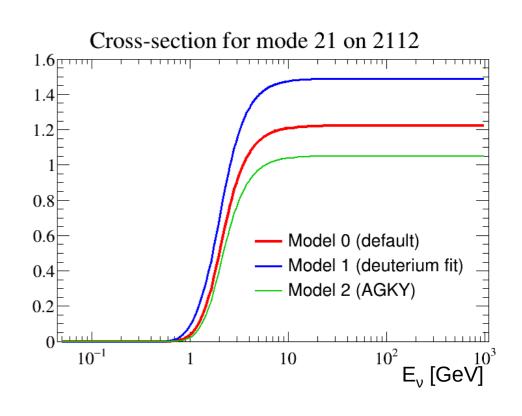
Non resonant background normalization Uncertainty on multiplicity model

Multi-pi mode cross-section is obtained by multiplying the total DIS cross section for W<2 GeV by the fraction of events that have at least two pions

Neutrino on proton

Cross-section for mode 21 on 2212 0.8 0.7 0.6 0.50.4 Model 0 (default) 0.3 Model 1 (deuterium fit) 0.2 Model 2 (AGKY) 0.1 10^{-1} 10^{2} 10 E_{ν} [GeV]

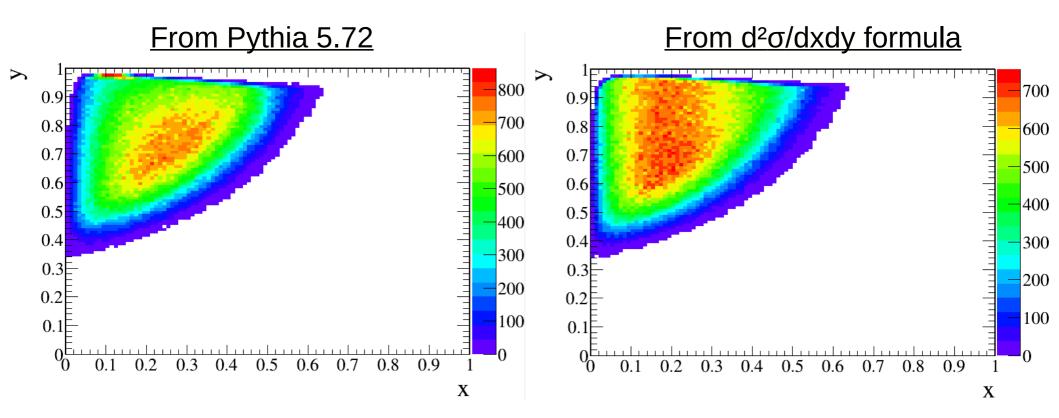
Neutrino on neutron



(In NEUT 5.4.0, cross-sections computed with model 0 are always used, regardless of the multiplicity model used)

DIS (PYTHIA) mode (x,y) generation

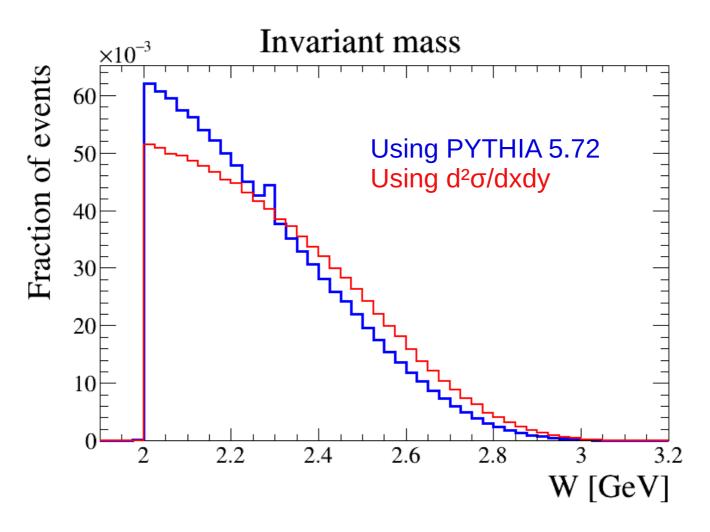
- Only provide neutrino and target nucleon four-momenta to PYTHIA Keep event if type (CC/NC) is the one desired, and W>2 GeV Redo PYTHIA generation with same inputs else
- As a result (x,y) distribution of events for PYTHIA events does not necessarily follow d²σ/dxdy



4 GeV v_{μ} on H_2O target

DIS (PYTHIA) mode W distribution

As a result, W distribution obtained for this W>2 GeV mode differ from the distribution we expect from the standard $d^2\sigma/dxdy$ (used for multi-pion mode)



4 GeV ν_{μ} on H_2O target, normalized by area

DIS (PYTHIA) mode

At last year's NuSTEC workshop on SIS/DIS region, one of the PYTHIA author warned us about using PYTHIA at "low" W

"I would not trust PYTHIA for anything with less than 6 pions"

Physics assumptions/limitations:

Always want to confine previously deconfined color.

Target-m not really present in x-section or q/g kinematics.

Only tested for W>4 GeV, small W in $e^+e^-\to h$ only, last global overview in 1987?

"Jet joining" not well-understood for low hadron multiplicity.

Strong isospin not traced in string.

Strings are traditionally non-interaction.

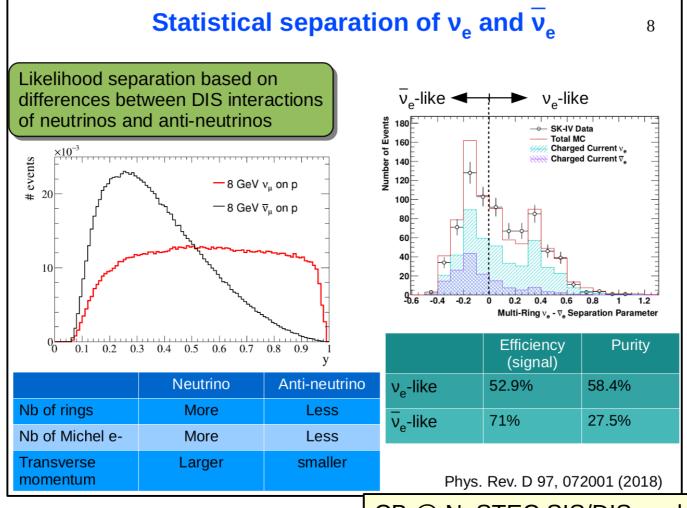
S. Prestel, "The LUND hadronization model"



Not clear the W distribution for the DIS/PYTHIA mode is reliable

Importance of hadronic system

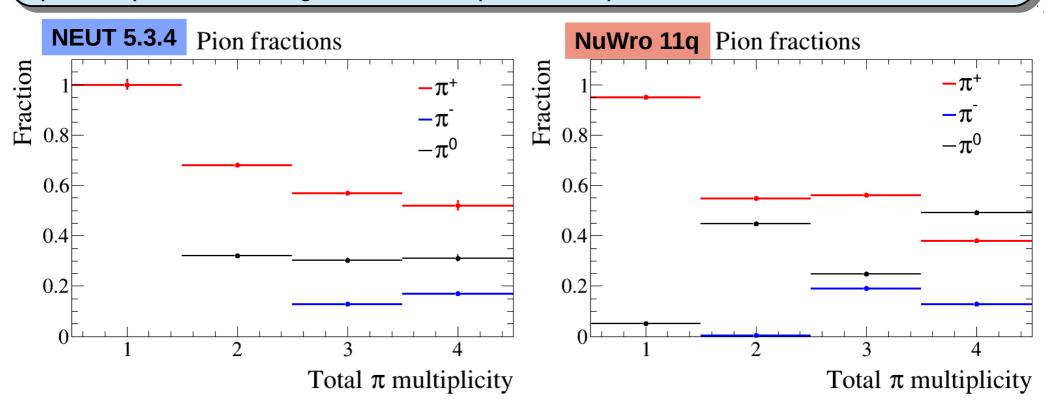
- NEUT is mainly used for Super-K (including T2K)
- In a water Cerenkov detector, different kind of pions look different (+ Cerenkov threshold for charged pions: appear as ring or Michel electrons depending on momentum)
- Topology of the event can change quite a bit depending on how the hadronic system is generated, can matter when trying to separate events with neural networks/deep learning



CB @ NuSTEC SIS/DIS workshop

Multi-pion mode Hadronic system

- Once number of hadrons is determined, generations of hadronic system is random-based for multi-pion mode:
 - 1 outgoing baryon (50% neutron, 50% proton)
 - remaining hadrons are pions (35% π^+ , 35% π^- , 30% π^0)
 - keep the event if charge is balanced, try again else
- Found that obtained pion fractions can be different from what PYTHIA 6 would predict (as seen throught the NuWro predictions)



Old comparison (from NuINT 2015), v_u on free protons, no FSI, 1.7<W<2.0 GeV

Summary

- > 2 modes producing pions in the resonance region for NEUT 5.4.0:
 - resonant model based on Rein-Sehgal for single pion production
 - custom DIS mode for multi-pion production
- Planning to include MK and DCC models for single pion production in the future
- Multi-pion mode need some work on NC part, and for improved modelization of the hadronic system
- Difficulties when trying to get all the modes together in a coherent way
- Current approach relies on separation between RES and DIS based on multiplicities, but large uncertainties on multiplicity model

BACKUP

Non resonant background Updates

List of updates NEUT 5.3.4 to 5.4.0:

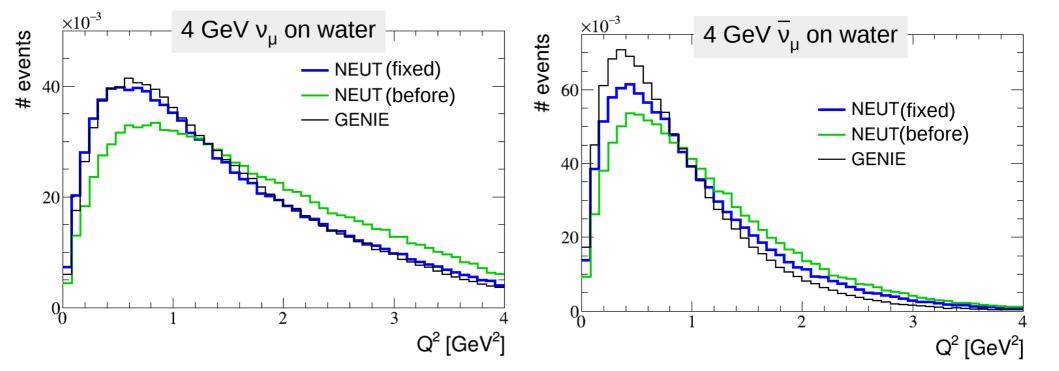
- Fix relation between Q^2 and x (avoid double counting target mass corrections)
- Use different cross sections to generate kinematics of interactions on protons and on neutrons
- Fix a typo in the implementation of the Bodek-Yang corrections
- target nucleon is selected based on the ratio of cross-sections on proton and neutron at the interaction energy
- Update version of the Bodek-Yang corrections used hep-ex/0301036 → hep-ph/0508007
- Change the scaling variable to the Nachtmann variable when Bodek-Yang corrections are not used
- Separate structure functions between CC and NC (still need to put the right formula for NC events)
- Use CKM matrix elements when calculating structure functions from PDF
- Added the charm related CKM matrix elements in the calculation of the structure functions if W is large enough to produce charmed particles (enough to produce a proton + a D0)
- Updated values of the CKM matrix elements to PDG 2015
- Take into account effect of multiplicity on generation of x and y
- Added possibility to use different hadron multiplicity model:
 - current NEUT one (with a small fix of the parameters)
 - my fit of deuterium bubble chamber data
 - AGKY model (used in GENIE)
- Removed passage through a Δ resonance

Low W model Fix relation between x and Q²

In the NEUT code, the definition of x used to obtain Q² was not the standard one:

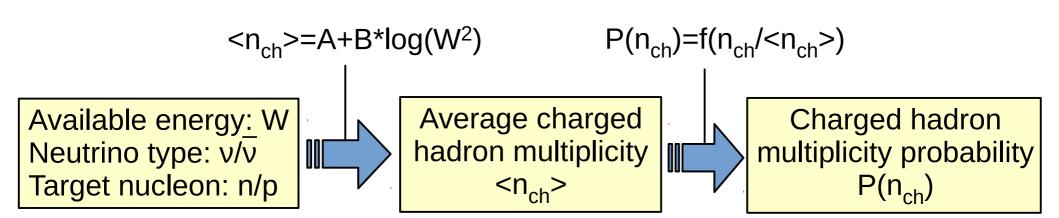
$$x = \frac{Q^2}{2M_{nuc}Ey}$$
 (1) but instead:
$$x = \frac{Q^2}{2M_{nuc}Ey + M_{nuc}^2}$$
 (2)

• It seems this is an old definition to take into account target mass effects, but they are already taken into account in corrections by Bodek and Yang In 5.3.4, x was generated assuming definition (1) but then (2) was used to deduce Q²



Multiplicity models (Hadronization for low W mode)

- Multiplicity models give the probability to produce a given number of hadrons for a given value of W
- > Based on KNO scaling: the distribution of $P(n_{ch})=f(n_{ch}/<n_{ch}>)$ is independent of W
- Average charged hadron multiplicity observed to be a linear fonction of log(W²) in bubble chamber data
 (K. Kuzmin and V. Naumov argue for a quadratic function at low W in PRC 88, 065501 (2013))



3 or 4 parameters for each couple of neutrino type and target nucleon depending on choice of f